

Résolution d'équations trigonométrique

Soient a donné et x une inconnue

● **1^{ère} cas :** $\begin{cases} \cos x = \cos a \\ \sin x = \sin a \end{cases} \Rightarrow \boxed{x = a + 2k\pi} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = \frac{1}{2} \\ \sin x = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{3} \\ \sin x = \sin \frac{\pi}{3} \end{cases} \Rightarrow \boxed{x = \frac{\pi}{3} + 2k\pi}$

● **2^{ème} cas :** $\begin{cases} \cos x = -\cos a \\ \sin x = \sin a \end{cases} \Rightarrow \boxed{x = \pi - a + 2k\pi} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = -\frac{\sqrt{2}}{2} \\ \sin x = \frac{\sqrt{2}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{4} \\ \sin x = \sin \frac{\pi}{4} \end{cases} \Rightarrow \boxed{x = \pi - \frac{\pi}{4} + 2k\pi = \frac{3\pi}{4} + 2k\pi}$

● **3^{ème} cas :** $\begin{cases} \cos x = \cos a \\ \sin x = -\sin a \end{cases} \Rightarrow \boxed{x = -a + 2k\pi} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = 0 \\ \sin x = -1 \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{2} \\ \sin x = -\sin \frac{\pi}{2} \end{cases} \Rightarrow \boxed{x = -\frac{\pi}{2} + 2k\pi}$

● **4^{ème} cas :** $\begin{cases} \cos x = -\cos a \\ \sin x = -\sin a \end{cases} \Rightarrow \boxed{x = \pi + a + 2k\pi} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = -\frac{1}{2} \\ \sin x = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{3} \\ \sin x = -\sin \frac{\pi}{3} \end{cases} \Rightarrow \boxed{x = \pi + \frac{\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi}$

Autres cas utiles

● $\cos x = \cos a$ $\Rightarrow x = 2k\pi + a$ ou $x = 2k\pi - a \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$		● $\sin x = \sin a$ $\Rightarrow x = 2k\pi + a$ ou $x = \pi - a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 2k\pi + \frac{\pi}{3}$ ou $x = \frac{2\pi}{3} + 2k\pi$
● $\cos x = -\cos a$ $\Rightarrow x = \pi + 2k\pi + a$ ou $x = \pi - a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} + 2k\pi$ ou $\frac{4\pi}{3} + 2k\pi$		● $\sin x = -\sin a$ $\Rightarrow x = -a + 2k\pi$ ou $x = \pi + a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = -\frac{\pi}{3} + 2k\pi$ ou $\frac{4\pi}{3} + 2k\pi$
● $\tan x = \tan a$ $\Rightarrow x = a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$	● $\tan x = -\tan a$ $\Rightarrow x = -a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = -1 \Rightarrow x = -\frac{\pi}{4} + k\pi$	● $\tan x = \frac{\tan a - \tan b}{1 + \tan a \tan b}$ $\Rightarrow x = a - b + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = \tan(a - b) \Rightarrow x = a - b + k\pi$
● $\cot x = \cot a$ $\Rightarrow x = a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$	● $\cot x = -\cot a$ $\Rightarrow x = -a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = -1 \Rightarrow x = -\frac{\pi}{4} + k\pi$	● $\cot x = \frac{\cot a \cot b - 1}{\cot b + \cot a}$ $\Rightarrow x = a + b + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = \cot(a + b) \Rightarrow x = a + b + k\pi$

Domaine de définition

- $\sin x$ défini sur $\mathbb{R}$
- $\cos x$ défini sur $\mathbb{R}$
- $\tan x$ défini sur $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$
- $\cot x$ défini sur $\mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$

Rappel

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \frac{1}{\cos^2 x} \\ 1 + \cot^2 x &= \frac{1}{\sin^2 x} \end{aligned}$$

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● **1^{ère} cas :** $\begin{cases} \cos x = \cos a \\ \sin x = \sin a \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{3} \\ \sin x = \sin \frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} x = a + 2k\pi \\ x = \frac{\pi}{3} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = \frac{1}{2} \\ \sin x = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{3} \\ \sin x = \sin \frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = \frac{\pi}{3} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

● **2^{ème} cas :** $\begin{cases} \cos x = -\cos a \\ \sin x = \sin a \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{4} \\ \sin x = \sin \frac{\pi}{4} \end{cases} \Rightarrow \begin{cases} x = \pi - a + 2k\pi \\ x = \frac{\pi}{4} + 2k\pi \\ x = \frac{3\pi}{4} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = -\frac{\sqrt{2}}{2} \\ \sin x = \frac{\sqrt{2}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{4} \\ \sin x = \sin \frac{\pi}{4} \end{cases} \Rightarrow \begin{cases} x = \pi - \frac{\pi}{4} + 2k\pi \\ x = \frac{\pi}{4} + 2k\pi \\ x = \frac{3\pi}{4} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

● **3^{ème} cas :** $\begin{cases} \cos x = \cos a \\ \sin x = -\sin a \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{2} \\ \sin x = -\sin \frac{\pi}{2} \end{cases} \Rightarrow \begin{cases} x = -a + 2k\pi \\ x = -\frac{\pi}{2} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = 0 \\ \sin x = -1 \end{cases} \Rightarrow \begin{cases} \cos x = \cos \frac{\pi}{2} \\ \sin x = -\sin \frac{\pi}{2} \end{cases} \Rightarrow \begin{cases} x = -\frac{\pi}{2} + 2k\pi \\ x = -\frac{\pi}{2} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

● **4^{ème} cas :** $\begin{cases} \cos x = -\cos a \\ \sin x = -\sin a \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{3} \\ \sin x = -\sin \frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} x = \pi + a + 2k\pi \\ x = \pi + \frac{\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

Exemple : $\begin{cases} \cos x = -\frac{1}{2} \\ \sin x = -\frac{\sqrt{3}}{2} \end{cases} \Rightarrow \begin{cases} \cos x = -\cos \frac{\pi}{3} \\ \sin x = -\sin \frac{\pi}{3} \end{cases} \Rightarrow \begin{cases} x = \pi + \frac{\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \\ x = \frac{4\pi}{3} + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$

Autres cas utiles

● $\cos x = \cos a$ $\Rightarrow x = 2k\pi + a$ ou $x = 2k\pi - a \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$	● $\sin x = \sin a$ $\Rightarrow x = 2k\pi + a$ ou $x = \pi - a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 2k\pi + \frac{\pi}{3}$ ou $x = \frac{2\pi}{3} + 2k\pi$
● $\cos x = -\cos a$ $\Rightarrow x = \pi + 2k\pi + a$ ou $x = \pi - a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$	● $\sin x = -\sin a$ $\Rightarrow x = -a + 2k\pi$ ou $x = \pi + a + 2k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = -\frac{\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$
● $\tan x = \tan a$ $\Rightarrow x = a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$	● $\tan x = -\tan a$ $\Rightarrow x = -a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = -1 \Rightarrow x = -\frac{\pi}{4} + k\pi$
● $\cot x = \cot a$ $\Rightarrow x = a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$	● $\cot x = -\cot a$ $\Rightarrow x = -a + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = -1 \Rightarrow x = -\frac{\pi}{4} + k\pi$
	● $\tan x = \frac{\tan a - \tan b}{1 + \tan a \tan b}$ $\Rightarrow x = a - b + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\tan x = \tan(a - b) \Rightarrow x = a - b + k\pi$
	● $\cot x = \frac{\cot a \cot b - 1}{\cot b + \cot a}$ $\Rightarrow x = a + b + k\pi \quad (k \in \mathbb{Z})$ <u>Exemple :</u> $\cot x = \cot(a + b) \Rightarrow x = a + b + k\pi$

Exercices

- $\cos x = \frac{\sqrt{2}}{2}$
- $\sin x = \frac{1}{2}$
- $\cos x = -\frac{1}{2}$
- $\sin x = -\frac{\sqrt{3}}{2}$
- $\tan x = 1$
- $\tan x = -1$
- $\cos x = \cos \frac{\pi}{6}$
- $\cos x = -\cos \frac{\pi}{4}$
- $\sin x = \sin \frac{\pi}{3}$
- $\sin x = -\sin \frac{\pi}{6}$
- $\cos x = 0$
- $\sin x = 1$
- $\tan x = \sqrt{3}$
- $\cot x = 1$
- $\cot x = -\sqrt{3}$
- $2 \sin x \cos x = \frac{\sqrt{3}}{2}$
- $\sin 2x = \frac{1}{2}$
- $\cos 2x = -\frac{1}{2}$
- $\tan(x + \frac{\pi}{4}) = 1$
- $\tan(x - \frac{\pi}{6}) = -\sqrt{3}$

Corrigés

- $x = 2k\pi \pm \frac{\pi}{4}$
- $x = 2k\pi + \frac{\pi}{6}$ ou $x = \frac{5\pi}{6} + 2k\pi$
- $x = \frac{2\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$
- $x = -\frac{\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$
- $x = \frac{\pi}{4} + k\pi$
- $x = -\frac{\pi}{4} + k\pi$
- $x = 2k\pi \pm \frac{\pi}{6}$
- $x = \frac{3\pi}{4} + 2k\pi$ ou $x = \frac{7\pi}{4} + 2k\pi$
- $x = 2k\pi + \frac{\pi}{3}$ ou $x = \frac{2\pi}{3} + 2k\pi$
- $x = -\frac{\pi}{6} + 2k\pi$ ou $x = \frac{7\pi}{6} + 2k\pi$
- $x = \frac{\pi}{2} + k\pi$
- $x = \frac{\pi}{2} + 2k\pi$
- $x = \frac{\pi}{3} + k\pi$
- $x = \frac{\pi}{4} + k\pi$
- $x = -\frac{\pi}{6} + k\pi$
- $x = \frac{\pi}{3} + k\pi$ ou $x = \frac{2\pi}{3} + k\pi$
- $x = \frac{\pi}{12} + k\pi$ ou $x = \frac{5\pi}{12} + k\pi$
- $x = \frac{\pi}{3} + k\pi$ ou $x = \frac{2\pi}{3} + k\pi$
- $x = k\pi$
- $x = -\frac{\pi}{3} + k\pi$

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Rappels

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