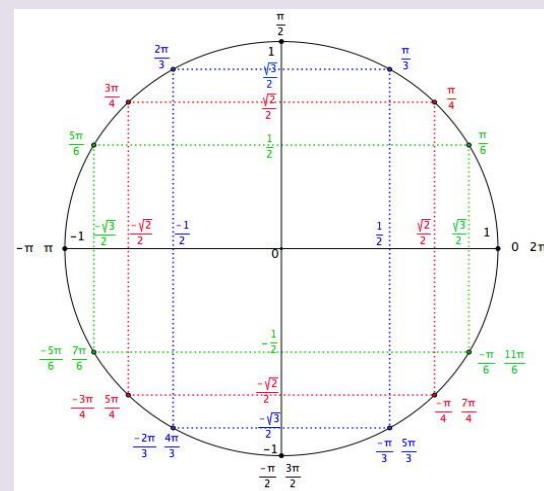


TRIGONOMETRIE

◆ *Valeurs remarquables :*

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		0



◆ *Les élémentaires:*

$\forall x \in \mathbb{R}$	$-1 \leq \cos x \leq 1$ $-1 \leq \sin x \leq 1$	$\cos^2 x + \sin^2 x = 1$
$\forall x \neq \frac{\pi}{2} + k\pi; k \in \mathbb{Z}$	$\tan x = \frac{\sin x}{\cos x}$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$

◆ *Angles associés à x :*

	Tour	
	$\cos(x + 2\pi) = \cos x$ $\sin(x + 2\pi) = \sin x$ $\tan(x + 2\pi) = \tan x$	
Angle opposé	Demi-tour	Quart de tour direct
$\cos(-x) = \cos x$ $\sin(-x) = -\sin x$ $\tan(-x) = -\tan x$	$\cos(x + \pi) = -\cos x$ $\sin(x + \pi) = -\sin x$ $\tan(x + \pi) = -\tan x$	$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$ $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ $\tan\left(x + \frac{\pi}{2}\right) = -\frac{1}{\tan x}$
Quart de tour indirect	Angle supplémentaire	Angle complémentaire
$\cos\left(x - \frac{\pi}{2}\right) = \sin x$ $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$ $\tan\left(x - \frac{\pi}{2}\right) = -\frac{1}{\tan x}$	$\cos(\pi - x) = -\cos x$ $\sin(\pi - x) = \sin x$ $\tan(\pi - x) = -\tan x$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\tan\left(\frac{\pi}{2} - x\right) = \frac{1}{\tan x}$

◆ *Formules d'addition :*

$\cos(a + b) = \cos a \cos b - \sin a \sin b$ $\cos(a - b) = \cos a \cos b + \sin a \sin b$	$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$
$\sin(a + b) = \sin a \cos b + \cos a \sin b$ $\sin(a - b) = \sin a \cos b - \cos a \sin b$	$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

◆ *Formules de duplication :*

$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$		$\cos^2 x = \frac{1 + \cos 2x}{2}$
$\sin 2x = 2 \sin x \cos x$	$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$	$\sin^2 x = \frac{1 - \cos 2x}{2}$

◆ *Transformation de Produits en Sommes :*

$\cos a \cos b = \frac{1}{2} [\cos(a - b) + \cos(a + b)]$	$\sin a \cos b = \frac{1}{2} [\sin(a - b) + \sin(a + b)]$
$\sin a \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$	$\cos a \sin b = -\frac{1}{2} [\sin(a - b) - \sin(a + b)]$

◆ *Transformation de Sommes en Produits :*

$\cos p + \cos q = 2 \cos\left(\frac{p + q}{2}\right) \cos\left(\frac{p - q}{2}\right)$	$\sin p + \sin q = 2 \sin\left(\frac{p + q}{2}\right) \cos\left(\frac{p - q}{2}\right)$
$\cos p - \cos q = -2 \sin\left(\frac{p + q}{2}\right) \sin\left(\frac{p - q}{2}\right)$	$\sin p - \sin q = 2 \sin\left(\frac{p - q}{2}\right) \cos\left(\frac{p + q}{2}\right)$

◆ *Equations trigonométriques :*

$\forall u \in \mathbb{R}, \forall v \in \mathbb{R}$	$\cos u = \cos v \Leftrightarrow \begin{cases} u = v + 2k\pi \\ u = -v + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$
	$\sin u = \sin v \Leftrightarrow \begin{cases} u = v + 2k\pi \\ u = \pi - v + 2k\pi \end{cases} \quad (k \in \mathbb{Z})$
$\forall u \neq \frac{\pi}{2} + k\pi, \forall v \neq \frac{\pi}{2} + k\pi$	$\tan u = \tan v \Leftrightarrow u = v + k\pi \quad (k \in \mathbb{Z})$

◆ *Equations particulières :*

$\cos t = 0 \Leftrightarrow t = \frac{\pi}{2} + k\pi$ $\sin t = 0 \Leftrightarrow t = k\pi$	$\cos t = -1 \Leftrightarrow t = \pi + 2k\pi$ $\sin t = -1 \Leftrightarrow t = -\frac{\pi}{2} + 2k\pi$
$\cos t = 1 \Leftrightarrow t = 2k\pi$ $\sin t = 1 \Leftrightarrow t = \frac{\pi}{2} + 2k\pi$	

◆ *Factorisation de $a \cos \omega x + b \sin \omega x$:*

Mettre $\sqrt{a^2 + b^2}$ en facteur

$$a \cos \omega x + b \sin \omega x = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \cos \omega x + \frac{b}{\sqrt{a^2 + b^2}} \sin \omega x \right)$$

Factorisation en cosinus

$$\text{Chercher } \alpha \in]-\pi ; \pi] / \begin{cases} \cos \alpha = \frac{a}{\sqrt{a^2 + b^2}} \\ \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}} \end{cases}$$

$$\text{On a alors : } a \cos \omega x + b \sin \omega x = \sqrt{a^2 + b^2} (\cos \alpha \cos \omega x + \sin \alpha \sin \omega x)$$

$$a \cos \omega x + b \sin \omega x = \sqrt{a^2 + b^2} \cos(\omega x - \alpha)$$

Factorisation en sinus

$$\text{Chercher } \beta \in]-\pi ; \pi] / \begin{cases} \sin \beta = \frac{a}{\sqrt{a^2 + b^2}} \\ \cos \beta = \frac{b}{\sqrt{a^2 + b^2}} \end{cases}$$

$$\text{On a alors : } a \cos \omega x + b \sin \omega x = \sqrt{a^2 + b^2} (\sin \beta \cos \omega x + \cos \beta \sin \omega x)$$

$$a \cos \omega x + b \sin \omega x = \sqrt{a^2 + b^2} \sin(\omega x + \beta)$$

◆ *Quelques résultats utiles :*

$\forall k \in \mathbb{Z},$	$\begin{cases} \cos(k\pi) = (-1)^k \\ \sin(k\pi) = 0 \end{cases}$	$\begin{cases} \sin\left(\frac{\pi}{2} + k\pi\right) = (-1)^k \\ \cos\left(\frac{\pi}{2} + k\pi\right) = 0 \end{cases}$
$\forall x \in \mathbb{R}, \forall k \in \mathbb{Z},$	$\begin{cases} \cos(x + k\pi) = (-1)^k \cos x \\ \sin(x + k\pi) = (-1)^k \sin x \end{cases}$	$\begin{cases} \cos(k\pi - x) = (-1)^k \cos x \\ \sin(k\pi - x) = -(-1)^k \sin x \end{cases}$