

Fonctions Trigonométriques et hyperboliques

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\operatorname{tg} \alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

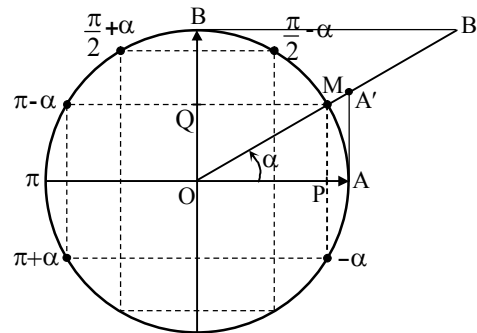
$$\cos \alpha = \overline{OP} \quad -1 \leq \cos \alpha \leq 1$$

$$\sin \alpha = \overline{OQ} \quad -1 \leq \sin \alpha \leq 1$$

$$\operatorname{tg} \alpha = \overline{AA'} = \frac{\sin \alpha}{\cos \alpha} = \frac{1}{\cot \alpha}$$

$$\cot \alpha = \overline{BB'} = \frac{\cos \alpha}{\sin \alpha} = \frac{1}{\operatorname{tg} \alpha}$$

$$\sec \alpha = \frac{1}{\cos \alpha} ; \operatorname{cosec} \alpha = \frac{1}{\sin \alpha}$$



$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha}$$

$$\cos(\alpha + 2k\pi) = \cos \alpha$$

$$\sin(\alpha + 2k\pi) = \sin \alpha$$

$$\operatorname{tg}(\alpha + k\pi) = \operatorname{tg} \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\sin(-\alpha) = -\sin \alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$$

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$$\cos(\pi + \alpha) = -\cos \alpha$$

$$\sin(\pi + \alpha) = -\sin \alpha$$

$$\operatorname{tg}(\pi + \alpha) = \operatorname{tg} \alpha$$

$$\cos(\pi - \alpha) = -\cos \alpha$$

$$\sin(\pi - \alpha) = \sin \alpha$$

$$\operatorname{tg}(\pi - \alpha) = -\operatorname{tg} \alpha$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

$$\operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\operatorname{tg} \alpha}$$

$$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$$

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$$

$$\operatorname{tg}\left(\frac{\pi}{2} + \alpha\right) = -\frac{1}{\operatorname{tg} \alpha}$$

$$\sin \alpha = 0 \Leftrightarrow \alpha = k\pi$$

$$\cos \alpha = 0 \Leftrightarrow \alpha = \frac{\pi}{2} + k\pi$$

$$\sin \alpha = 1 \Leftrightarrow \alpha = \frac{\pi}{2} + 2k\pi$$

$$\cos \alpha = 1 \Leftrightarrow \alpha = 2k\pi$$

$$\sin \alpha = -1 \Leftrightarrow \alpha = \frac{3\pi}{2} + 2k\pi$$

$$\cos \alpha = -1 \Leftrightarrow \alpha = \pi + 2k\pi$$

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$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1$$

$$= 1 - 2\sin^2 \alpha = \frac{1 - \operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha = \frac{2 \operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha}$$

$$2\cos^2 \alpha = 1 + \cos 2\alpha$$

$$2\sin^2 \alpha = 1 - \cos 2\alpha$$

$$4\cos^3 \alpha = \cos 3\alpha + 3\cos \alpha$$

$$4\sin^3 \alpha = -\sin 3\alpha + 3\sin \alpha$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta}$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta}$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\cos \alpha = \cos \beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \alpha = -\beta + 2k\pi \end{cases}$$

$$\sin \alpha = \sin \beta \Leftrightarrow \begin{cases} \alpha = \beta + 2k\pi \\ \alpha = \pi - \beta + 2k\pi \end{cases}$$

$$\operatorname{tg} \alpha = \operatorname{tg} \beta \Leftrightarrow \alpha = \beta + k\pi$$

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$$\cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

$$\sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i}$$

$$e^{i\alpha} = \cos \alpha + i \sin \alpha$$

$$e^{-i\alpha} = \cos \alpha - i \sin \alpha$$

$$e = 2,718281828...$$

$$i \text{ nbre imag: } i^2 = -1$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\operatorname{tg} \alpha \operatorname{tg} \beta = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\cot \alpha + \cot \beta} = \frac{\operatorname{tg} \beta - \operatorname{tg} \alpha}{\cot \alpha - \cot \beta}$$

$$a \cos \alpha + b \sin \alpha = \sqrt{a^2 + b^2} \sin(\alpha + \theta)$$

$$\sin \theta = \frac{a}{\sqrt{a^2 + b^2}} ; \cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$a \cos \alpha + b \sin \alpha = \sqrt{a^2 + b^2} \cos(\alpha - \theta)$$

$$\cos \theta = \frac{a}{\sqrt{a^2 + b^2}} ; \sin \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \sin\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$$

$$\operatorname{tg} \alpha - \operatorname{tg} \beta = \frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta}$$

$$\operatorname{ch} \alpha = \frac{e^\alpha + e^{-\alpha}}{2} ; \operatorname{sh} \alpha = \frac{e^\alpha - e^{-\alpha}}{2}$$

$$\operatorname{th} \alpha = \frac{\operatorname{sh} \alpha}{\operatorname{ch} \alpha} = \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}}$$

$$\operatorname{cth} \alpha = \frac{\operatorname{ch} \alpha}{\operatorname{sh} \alpha} = \frac{e^\alpha + e^{-\alpha}}{e^\alpha - e^{-\alpha}}$$

$$\operatorname{ch} \alpha + \operatorname{sh} \alpha = e^\alpha ; \operatorname{ch} \alpha - \operatorname{sh} \alpha = e^{-\alpha}$$

$$\operatorname{ch}^2 \alpha - \operatorname{sh}^2 \alpha = 1 ; \operatorname{th} \alpha \operatorname{cth} \alpha = 1$$

$$\operatorname{sh}(-\alpha) = -\operatorname{sh} \alpha ; \operatorname{ch}(-\alpha) = \operatorname{ch} \alpha$$

$$\operatorname{th}(-\alpha) = -\operatorname{th} \alpha ; \operatorname{cth}(-\alpha) = -\operatorname{cth} \alpha$$

$$\operatorname{sh}(\alpha \pm \beta) = \operatorname{sh} \alpha \operatorname{ch} \beta \pm \operatorname{ch} \alpha \operatorname{sh} \beta$$

$$\operatorname{ch}(\alpha \pm \beta) = \operatorname{ch} \alpha \operatorname{ch} \beta \pm \operatorname{sh} \alpha \operatorname{sh} \beta$$

$$\operatorname{th}(\alpha \pm \beta) = \frac{\operatorname{th} \alpha \pm \operatorname{th} \beta}{1 \pm \operatorname{th} \alpha \operatorname{th} \beta}$$

$$\operatorname{sh} 2\alpha = 2\operatorname{sh} \alpha \operatorname{ch} \alpha = \frac{2 \operatorname{th} \alpha}{1 - \operatorname{th}^2 \alpha}$$

$$\operatorname{ch} 2\alpha = \operatorname{ch}^2 \alpha + \operatorname{sh}^2 \alpha = 2\operatorname{sh}^2 \alpha + 1$$

$$= 2\operatorname{ch}^2 \alpha - 1 = \frac{1 + \operatorname{th}^2 \alpha}{1 - \operatorname{th}^2 \alpha}$$

$$\operatorname{th} 2\alpha = \frac{2 \operatorname{th} \alpha}{1 + \operatorname{th}^2 \alpha} ; \operatorname{cth} 2\alpha = \frac{1 + \operatorname{cth}^2 \alpha}{2 \operatorname{cth} \alpha}$$

$$\operatorname{sh} \alpha \pm \operatorname{sh} \beta = 2 \operatorname{sh}\left(\frac{\alpha \pm \beta}{2}\right) \operatorname{ch}\left(\frac{\alpha \mp \beta}{2}\right)$$

$$\operatorname{ch} \alpha + \operatorname{ch} \beta = 2 \operatorname{ch}\left(\frac{\alpha + \beta}{2}\right) \operatorname{ch}\left(\frac{\alpha - \beta}{2}\right)$$

$$\operatorname{ch} \alpha - \operatorname{ch} \beta = 2 \operatorname{sh}\left(\frac{\alpha + \beta}{2}\right) \operatorname{sh}\left(\frac{\alpha - \beta}{2}\right)$$

$$\operatorname{th} \alpha \pm \operatorname{th} \beta = \frac{\operatorname{sh}(\alpha \pm \beta)}{\operatorname{ch} \alpha \operatorname{ch} \beta}$$

$$(\operatorname{ch} \alpha \pm \operatorname{sh} \alpha)^n = \operatorname{ch} n\alpha \pm \operatorname{sh} n\alpha$$