

# DEMONTRER EN UTILISANT LE RAISONNEMENT PAR RECURRENTE QUE :

- 1)  $\forall n \in \mathbb{N}^* : 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
- 2)  $\forall n \in \mathbb{N}^* : 1 + 2 + 3 + \dots + (2n-1) = n^2$
- 3)  $\sum_{K=1}^n K(K+1)(K+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- 4)  $\sum_{K=1}^n K2^{K-1} = (n-1)2^n + 1$
- 5)  $\sum_{K=1}^n (2K) = n(n+1)$
- 6)  $\sum_{K=1}^n (2K-1) = n^2$
- 7)  $\sum_{K=1}^n K^2 = \frac{n(n+1)(2n+1)}{6}$
- 8)  $\sum_{K=1}^n K^3 = \frac{n^2(n+1)^2}{4}$
- 9)  $\sum_{K=1}^n K^3 = (1+2+\dots+n)^2$
- 10)  $\sum_{K=1}^n K^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- 11)  $\sum_{K=1}^n K = \frac{n(n+1)}{2}$
- 12)  $\sum_{K=1}^n K(n-K) = \frac{(n-1)n(n+1)}{6}$
- 13)  $\sum_{K=1}^n K^2(n-K) = \frac{n^2(n+1)(n-1)}{12}$
- 14)  $\sum_{K=2}^n (K-1)2^{K-2} = (n-1)2^n - n2^{n-1} + 1$
- 15)  $\sum_{K=1}^n K(K+1)(K+2) = \frac{n(n+1)(n+2)(n+3)}{3}$
- 16)  $\sum_{K=1}^n K(K+1) = \frac{n(n+1)(n+2)}{3}$
- 17)  $\sum_{K=1}^n K^2(n^3 - K^2) = \frac{n(n+1)(2n+1)(n-1)(5n^2+2n-1)}{30}$
- 18)  $\sum_{K=1}^n (K^2 - 1) = \frac{n(n-1)(2n+5)}{3}$
- 19)  $\sum_{K=1}^n K(n^2 - K^2) = \frac{n^2(n-1)(n+1)}{4}$
- 20)  $\sum_{K=1}^n \frac{1}{4K^2-1} = \frac{n}{2n+1}$
- 21)  $\sum_{K=1}^n (1 - K^3) = \frac{n(1-n)(n^2+3n+4)}{4}$
- 22)  $\sum_{K=0}^n (K+1)X^K = \frac{1-(n+2)X^{n+1}+(n+1)X^{n+2}}{(1-X)^2}$
- 23)  $\sum_{K=1}^n \frac{1}{K(K+1)} = \frac{n}{n+1}$
- 24)  $\sum_{K=1}^n \frac{1}{K(K+1)(K+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$
- 25)  $\sum_{K=1}^n \frac{2K-1}{2^K} = 3 - \frac{3+2n}{2^n}$
- 26)  $\sum_{K=0}^n \left( \frac{7-3^{K+1}}{2 \cdot 3^K} \right) = -\frac{3}{2}(n+1) + \frac{21}{4} - \frac{7}{4 \cdot 3^n}$
- 27)  $\sum_{K=1}^n X^K = \frac{1-X^{n+1}}{1-X}$
- 28)  $\sum_{K=1}^n K \cdot K! = (n+1)! - 1$
- 29)  $\sum_{K=0}^n \left( \frac{2}{3 \cdot 2^K} + \frac{1}{\sqrt{2}} K \right) = \frac{4}{3} - \frac{1}{3 \cdot 2^{n-1}} + \frac{n(n+1)}{2\sqrt{2}}$
- 30)  $\prod_{K=2}^n \left( 1 - \frac{1}{K^2} \right) = -\frac{n+1}{2n}$
- 31)  $\sum_{K=1}^n \sin(2K-1)X = \frac{\sin^2(nx)}{\sin X}$
- 32)  $\sum_{K=1}^n K \cdot K! = (n+1) - 1$
- 33)  $\sum_{K=1}^n \sqrt{2} \cos \left[ (2K-1)X - \frac{\pi}{4} \right] = \frac{\sqrt{2} \sin(nx) \cos(nx - \frac{\pi}{4})}{\sin x}$
- 34)  $\sum_{K=1}^n \frac{K}{(K+1)!} = 1 - \frac{1}{(n+1)!}$
- 35)  $\sum_{K=1}^n K \cdot X^K = \frac{nx^{n+2} - (n+1)x^{n+1} + x}{(1-x)^2}$
- 36)  $\sum_{K=1}^n K(-1)^K = \frac{(-1)^n(2n+1)-1}{4}$
- 37)  $\sum_{K=1}^n K \cdot 5^K = \frac{5+5^{n+1}(4n-1)}{16}$
- 38)  $\sum_{K=1}^n \frac{K}{(K+1)!} = 1 - \frac{1}{(n+1)!}$
- 39)  $\sum_{K=1}^n \cos(2K-1)X = \cos(nx) \frac{\sin(nx)}{\sin x}$
- 40)  $\sum_{K=1}^n (2K+1)(-1)^K = (n+1)(-1)^n$
- 41)  $\sum_{K=1}^n K(K+2) = \frac{n(n+1)-2n+1}{6}$
- 42)  $\sum_{K=1}^n 2K(-1)^{K-1} = \frac{1-(2n+1)(-1)^n}{2}$
- 43)  $\sum_{K=1}^n \frac{K}{2^K} = 2 - \frac{n+2}{2^n}$
- 44)  $\sum_{K=1}^n (-1)^K \cdot K = \frac{(-1)^n(n+1)-1}{4}$
- 45)  $\sum_{K=1}^n (K^2 + 1)K! = n(n+1)!$
- 46)  $\sum_{K=1}^n \frac{1}{(2K-1)(2K+1)} = \frac{n}{2n+1}$
- 47)  $\sum_{K=1}^n \frac{1}{(3K-2)(3K+1)} = \frac{n}{3n+1}$
- 48)  $\sum_{k=1}^n \frac{1}{\log_k^x} = \frac{1}{\log_n^x}$
- 49)  $\sum_{k=1}^n \log \frac{n}{k} = \log \left( \frac{n^n}{n!} \right)$
- 50)  $\sum_{k=1}^n K^2 \cdot (-1)^{K-1} = (-1)^{n-1} \sum_{k=1}^n k$
- 51)  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
- 52)  $\sum_{K=0}^n 3^K = \frac{1}{2}(3^{n+1} - 1)$
- 53)  $\sum_{k=0}^n q^k = \frac{q^{n+1}-1}{q-1}$
- 54)  $\sum_{k=1}^n K \left( \frac{4}{5} \right)^k = \frac{4 \times 5^{n+1} - (5+n) \times 4^{n+1}}{5^n}$
- 55)  $\prod_{k=0}^n \cos(2^k X) = \frac{1}{2^{n+1}} \times \frac{\sin(2^{n+1} X)}{\sin X}$
- 56)  $\forall n \geq 0 : 3^{3n+2} + 2^{n+4} \text{ est divisible par } 5$
- 57)  $\forall n \geq 1 : 3^{2n} - 2^n \text{ est divisible par } 7$