

Integration par partie

$$\int_a^b f(x) g(x) dx$$

$$\frac{1}{2} \times x$$

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posons $u = \dots \rightarrow u' = \dots$
 $v' = \dots \rightarrow v = \dots$

$$\int_a^b f(x) g(x) dx = [u \times v]_a^b - \int_a^b v \times u' dx$$

on pose $u \Rightarrow$ L.P.E.S

ln poly expo sin cos tan

Exercice

Calcule les intégrales suivantes

$$I = \int_1^e x \ln x \, dx; \quad I = \int_1^e \ln x \cdot dx$$

$$I = \int_1^e \frac{\ln x}{x^2} \, dx; \quad I = \int_1^e \frac{\ln x}{x} \, dx (*)$$

$$I = \int_0^1 x e^x \, dx; \quad I = \int_0^{\pi} x \sin x \, dx$$

$$I = \int_0^{\frac{\pi}{2}} x \cos x \, dx; \quad I = \int_0^1 x \sqrt{3+x} \, dx$$

$$I = \int_0^{\frac{\pi}{2}} (2x+1) \cos 2x \, dx; \quad I = \int_1^e x^2 \ln x \, dx$$

Solution

$$[F(x)]_a^b = F(b) - F(a)$$

$$\ln e = 1$$
$$\ln 1 = 0$$

$$I = \int_1^e x \ln x \, dx$$

posons

$$u = \ln x \Rightarrow u' = \frac{1}{x}$$

$$v' = x \Rightarrow v = \frac{x^2}{2}$$

(3)

$$I = \left[\frac{x^2}{2} \ln x \right]_1^e - \int_1^e \left(\frac{x^2}{2} \times \frac{1}{x} \right) dx$$

$$= \left[\frac{x^2}{2} \ln x \right]_1^e - \int_1^e \frac{x}{2} dx$$

$$= \left[\frac{x^2}{2} \ln x \right]_1^e - \left[\frac{x^2}{4} \right]_1^e$$

$$= \left[\frac{x^2}{2} \ln x \right]_1^e - \left[\frac{x^2}{4} \right]_1^e$$

$$= \left[\frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1$$

$$= \left(\frac{e^2}{2} \ln e - \frac{e^2}{4} \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right)$$

$$= \left(\frac{e^2}{2} - \frac{e^2}{4} \right) - \left(-\frac{1}{4} \right)$$

$$= \frac{2e^2 - e^2}{4} + \frac{1}{4}$$

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$$\boxed{I = \frac{e^2}{4} + \frac{1}{4}} \quad \text{ou}$$

$$\boxed{I = \frac{e^2 + 1}{4}}$$

$$I = \int_1^e \frac{1}{x} \ln x \, dx$$

$$\frac{1}{x^n} \rightarrow \frac{-1}{(n-1)x^{n-1}}$$

posons. $u = \ln x \Rightarrow u' = \frac{1}{x}$

$v' = 1 \Rightarrow v = x$

$$I = [x \ln x]_1^e - \int_1^e 1 \, dx$$

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$$= [x \ln x]_1^e - [x]_1^e$$

$$= [x \ln x - x]_1^e$$

$$= (e \ln e - e) - (1 \ln 1 - 1)$$

$$= 0 - (-1)$$

$$\boxed{I = 1}$$

$$I = \int_1^e \frac{\ln x}{x^2} dx = \int_1^e \frac{1}{x^2} \cdot \ln x dx$$

posons $u = \ln x \Rightarrow u' = \frac{1}{x}$

$$v' = \frac{1}{x^2} \Rightarrow v = -\frac{1}{x}$$

⑥

$$I = \left[-\frac{1}{x} \ln x \right]_1^e - \int_1^e -\frac{1}{x^2} dx$$

$$= \left[-\frac{1}{x} \ln x \right]_1^e + \left[\frac{1}{x} \right]_1^e$$

$$= \left[-\frac{1}{x} \ln x - \frac{1}{x} \right]_1^e$$

$$= \left(-\frac{1}{e} \ln e - \frac{1}{e} \right) - \left(-\ln 1 - 1 \right)$$

$$\boxed{I = \frac{2}{e} + 1}$$

$$I = \int_0^1 x e^x dx.$$

⑦

posons.

$$u = x \Rightarrow u' = 1$$

$$v' = e^x \Rightarrow v = e^x$$

$$I = [x e^x]_0^1 - \int_0^1 e^x dx.$$

$$= [x e^x]_0^1 - [e^x]_0^1$$

$$= [x e^x - e^x]_0^1$$

$$= (1e - e) - (0e^0 - e^0)$$

$$= 0 - (0 - 1)$$

$$\boxed{I = 1}$$

$$I = \int_0^{\pi} x \sin x \, dx \quad x(-\cos x)$$

posons $u = x \rightarrow u' = 1$

$$v' = \sin x \rightarrow v = -\cos x.$$

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$$I = \left[-x \cos x \right]_0^{\pi} - \int_0^{\pi} -\cos x \, dx.$$

$$= \left[-x \cos x \right]_0^{\pi} + \left[\sin x \right]_0^{\pi}$$

$$= \left[-x \cos x + \sin x \right]_0^{\pi}$$

$$= (-\pi \cos \pi + \sin \pi) - (0 \cos 0 + \sin 0)$$

$$= (\pi + 0) - (0 + 0)$$

$$\boxed{I = \pi}$$

$$f(x) = \sqrt{3+x}$$

$$u = 3+x$$

$$u' = 1$$

$$F(x) = \frac{2}{3} (3+x) \sqrt{3+x}$$

$$I = \int_0^{\frac{\pi}{2}} x \cos x \, dx.$$

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posons $u = x \Rightarrow u' = 1$

$v' = \cos x \Rightarrow v = \sin x$

$$I = \left[x \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx.$$

$$= \left[x \sin x \right]_0^{\frac{\pi}{2}} - \left[-\cos x \right]_0^{\frac{\pi}{2}}$$

$$= \left[x \sin x + \cos x \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} \right) - (0 \sin 0 + \cos 0)$$

$$= \left(\frac{\pi}{2} + 0 \right) - (0 + 1)$$

$$\boxed{I = \frac{\pi}{2} - 1}$$

$$I = \int_0^1 x \sqrt{3+x} dx$$

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posons $u = x \Rightarrow u' = 1$

$$v' = \sqrt{3+x} \Rightarrow v = \frac{2}{3}(3+x)\sqrt{3+x}$$

$$I = \left[\frac{2}{3} x(3+x)\sqrt{3+x} \right]_0^1 - \int_0^1 \frac{2}{3}(3+x)\sqrt{3+x} dx$$

$$= \left[\frac{2}{3} x(3+x)\sqrt{3+x} \right]_0^1 - \int_0^1 \frac{2}{3} (3\sqrt{3+x} + x\sqrt{3+x}) dx$$

$$= \left[\frac{2}{3} x(3+x)\sqrt{3+x} \right]_0^1 - \int_0^1 \left(2\sqrt{3+x} + \frac{2}{3} x\sqrt{3+x} \right) dx$$

$$I = \left[\frac{2}{3} x(3+x)\sqrt{3+x} \right]_0^1 - \int_0^1 2\sqrt{3+x} dx - \int_0^1 \frac{2}{3} x\sqrt{3+x} dx$$

$$I = \left[\frac{2}{3} x(3+x)\sqrt{3+x} \right]_0^1 - \left[2 \times \frac{2}{3} (3+x)\sqrt{3+x} \right]_0^1 - \frac{2}{3} I$$

$$I + \frac{2}{3}I = \left[\frac{2}{3}x(3+x)\sqrt{3+x} - \frac{4}{3}(3+x)\sqrt{3+x} \right]_0^1$$

$$\frac{5}{3}I = \left(\frac{2}{3}(4)(2) - \frac{4}{3}(4)(2) \right) - \left(0 - \frac{4}{3}(3)\sqrt{3} \right)$$

$$\frac{5}{3}I = \frac{16}{3} - \frac{32}{3} + 4\sqrt{3}$$

$$\frac{5}{3}I = -\frac{16}{3} + 4\sqrt{3}$$

$$5I = -16 + 12\sqrt{3}$$

$$I = \frac{-16 + 12\sqrt{3}}{5}$$

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$$I = \int_0^{\frac{\pi}{2}} (2x+1) \cos 2x \, dx$$

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posons $u = 2x+1 \Rightarrow u' = 2$

$$v' = \cos 2x \Rightarrow v = \frac{1}{2} \sin 2x$$

$$I = \left[\frac{1}{2} (2x+1) \sin 2x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin 2x \, dx$$

$$= \left[\frac{1}{2} (2x+1) \sin 2x \right]_0^{\frac{\pi}{2}} - \left[-\frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{1}{2} (2x+1) \sin 2x + \frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{1}{2} (\pi+1) \sin \pi + \frac{1}{2} \cos \pi \right) - \left(\frac{1}{2} (1) \sin 0 + \frac{1}{2} \cos 0 \right)$$

$$= \left(0 - \frac{1}{2} \right) - \left(\frac{1}{2} \right)$$

$$I = -1$$

$$I = \int_1^e x^2 \ln x \cdot dx$$

posons $u = \ln x \Rightarrow u' = \frac{1}{x}$
 $v' = x^2 \Rightarrow v = \frac{x^3}{3}$

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$$I = \left[\frac{x^3}{3} \ln x \right]_1^e - \int_1^e \frac{x^2}{3} dx$$

$$= \left[\frac{x^3}{3} \ln x \right]_1^e - \left[\frac{x^3}{9} \right]_1^e$$

$$= \left[\frac{x^3}{3} \ln x - \frac{x^3}{9} \right]_1^e$$

$$= \left(\frac{e^3}{3} \ln e - \frac{e^3}{9} \right) - \left(\frac{1}{3} \ln 1 - \frac{1}{9} \right)$$

$$I = \frac{2e^3}{9} + \frac{1}{9}$$

$$I = \int_1^e \frac{\ln x}{x} dx = \int_1^e \frac{1}{x} \ln x dx$$

soit $f(x) = \frac{1}{x} \ln x$

$$u = \ln x \rightarrow u' = \frac{1}{x}$$

$$F(x) = \frac{(\ln x)^2}{2}$$

$$\begin{cases} F(e) = \frac{(\ln e)^2}{2} = \frac{1}{2} \\ F(1) = \frac{(\ln 1)^2}{2} = 0 \end{cases}$$

$$I = \frac{1}{2}$$

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